Exercise 10

Differentiate both sides of the following equations:

$$1 + xe^x = \int_0^x e^{x-t} u(t)\, dt$$

Solution

Differentiating both sides of the equation with respect to x gives us

$$e^x + xe^x = u(x) \cdot 1 - e^x u(0) \cdot 0 + \int_0^x \frac{\partial}{\partial x} e^{x-t} u(t)\, dt,$$

where we used the Leibnitz rule to differentiate the integral. Therefore,

$$e^x(x+1) = u(x) + \int_0^x e^{x-t} u(t)\, dt.$$